

Magnetar Spin-Down

Alice K. Harding, Ioannis Contopoulos and Demosthenes Kazanas

Laboratory for High Energy Astrophysics

NASA/Goddard Space Flight Center

Greenbelt, MD 20771

ABSTRACT

We examine the effects of a relativistic wind on the spin down of a neutron star and apply our results to the study of Soft Gamma Repeaters (SGRs), thought to be neutron stars with magnetic fields $> 10^{14}$ G. We derive a spin-down formula that includes torques from both dipole radiation and episodic or continuous particle winds. We find that if SGR1806-20 puts out a continuous particle wind of 10^{37} erg s $^{-1}$, then the pulsar age is consistent with that of the surrounding supernova remnant, but the derived surface dipole magnetic field is only 3×10^{13} G, in the range of normal radio pulsars. If instead, the particle wind flows are episodic with small duty cycle, then the observed period derivatives imply magnetar-strength fields, while still allowing characteristic ages within a factor of two of the estimated supernova remnant age. Close monitoring of the periods of SGRs will be able to establish or place limits on the wind duty cycle and thus the magnetic field and age of the neutron star.

Subject headings: stars: neutron, winds; gamma rays: bursts; magnetic fields

1. INTRODUCTION

There is a growing collection of pulsating high-energy sources which occupy a unique phase-space in their combination of long (> 5 s), monotonically increasing periods and high period derivatives. One subgroup in this collection are the soft gamma-ray repeaters (SGRs), transient sources that exhibit repeated bursts of soft (~ 30 keV), short duration (0.1 s) γ -rays. Bursts of average energy 10^{41} ergs repeat on irregular intervals, while giant bursts of energy $\sim 10^{45}$ ergs have been observed once in each of the sources SGR 0526-66 and SGR 1900+14. Recently, a period of $P = 7.47$ s has been detected from SGR 1806-20 in quiescent emission (Kouveliotou et al. 1998) and a period of $P = 5.16$ s from SGR 1900+14 in both quiescent (Kouveliotou et al. 1999) and giant burst emission (Hurley et al. 1999a).

Both have measured period derivatives around $\dot{P} \sim 10^{-10} \text{ s s}^{-1}$. Another group of sources having similar P and \dot{P} are the anomalous X-ray pulsars (AXPs), pulsating X-ray sources with periods in the range 6 – 12 s and period derivatives in the range $10^{-12} - 10^{-11} \text{ s s}^{-1}$ (Gotthelf & Vasisht 1998). These sources have shown only quiescent emission with no bursting behavior.

The most plausible and commonly invoked model to explain the characteristics of both the SGRs and AXPs is that of a neutron star having a dipole magnetic field of $> 10^{14}$ G, much higher than the fields of ordinary, isolated pulsars and well above the quantum critical field of 4.4×10^{13} G. Such stars, known as magnetars, were first proposed by Duncan & Thompson (1992), Usov (1992) and Paczynski (1992) to account for various properties of SGRs and γ -ray bursts. All of the SGR and AXP sources are found in or near young ($\tau < 10^5$ yr) supernova remnants (SNR). SGRs in addition have been associated with X-ray and radio plerions whose emission power far exceeds the dipole spin-down power. It was therefore proposed that relativistic particle outflows from the SGR bursts (Tavani 1994, Harding 1995, Frail et al. 1997) or from a steady flux of Alfvén waves (Thompson & Duncan 1996), provide the power for the nebular emission. The existence of such a wind has been inferred indirectly by X-ray and radio observations of the synchrotron nebula G10.0–0.3 around SGR1806–20 (Murakami et al. 1994, Kulkarni et al. 1994)¹. Thompson & Duncan (1996) estimated that the particle luminosity from SGR1806–20 is of the order of $10^{37} \text{ erg s}^{-1}$. Such energetic particle winds will also affect the spin-down torque of the star by distorting the dipole field structure near the light cylinder (Thompson & Blaes 1998 [TB98]). We will show, however, deriving a formula similar to the one given by TB98, that if relativistic wind outflow continuously dominates the spin-down of the neutron star in SGR 1806-20 at a level $\gtrsim 10^{36} \text{ erg s}^{-1}$, then the surface magnetic dipole field is too low to be consistent with a magnetar model. It is possible however that the wind outflows from SGR sources are episodic, lasting for a time following each burst that may be small compared to the time between bursts. In this case, dipole radiation will dominate the spin-down between bursts, even though wind outflow may dominate the average rotational energy-loss rate. Using a general formula for the spin-down torque that includes both dipole radiation and episodic particle winds, we derive the magnetic field and characteristic age of the neutron star as a function of the observed P and \dot{P} , the wind luminosity and wind duty cycle. We find that the derived surface field and age can have a range of values between the pure dipole and pure wind cases that depend on the duty cycle of the wind outflows, even for a constant value of the average wind luminosity.

¹Note that a revised IPN location now places the SGR source outside the core of the radio plerion (Hurley et al. 1999b)

2. ENERGY LOSS FROM A WIND

It is known that neutron star rotation drastically distorts the magnetic field near and beyond the light cylinder radius $R_{LC} = c/\Omega$ (where $\Omega = 2\pi/P$ is the neutron star angular velocity; P is the period), and that magnetic field lines crossing the light cylinder remain open all the way to ‘infinity’. Field lines will also open up in the presence of a powerful wind of particles emitted from the neutron star. Near the surface of the star, dipole magnetic field pressure is high enough to completely dominate the wind stresses. However, the magnetic energy density drops much faster with distance than that of a quasi-isotropic particle wind of kinetic luminosity L_p at infinity; thus, ignoring a transitional region between these two regimes, beyond a distance of the order $r_{\text{open}} \sim r_0(B_0^2 r_0^2 c / 2L_p)^{1/4}$ magnetic field lines will be ‘combed out’ by the wind. Here, B_0 is the value of the neutron star surface dipole magnetic field and $r_0 \sim 10$ km, is the radius of the star (henceforth, we will use small r to denote spherical distances [from the center], and capital R for cylindrical distances [from the axis]). The fraction of open field lines originates from a region of radial extent

$$R_{pc} \approx r_0(r_0/r_{\text{open}})^{1/2} \ll r_0 \quad (1)$$

around the axis, the so-called polar cap (this estimate is obtained for an undistorted dipole).

An aligned magnetic rotator, the simplified geometry examined herein, spins down (even though an aligned magnetic rotator in vacuum *does not*), because a neutron star magnetosphere *cannot be* a true vacuum (Goldreich & Julian 1969). As is discussed in Contopoulos, Kazanas & Fendt (1999; hereafter CKF), a neutron star magnetosphere is spontaneously charged in order to support the charges and electric currents required in the realistic solution. An electric current also flows, in a large scale electric circuit through the polar cap to infinity, closing along an equatorial current sheet discontinuity, which connects to the interface between open and closed field lines at the light cylinder, with the circuit closing *across* field lines along the polar cap. This electric current I_{pc} flows along the magnetic field lines crossing the polar cap and generates the required spin-down magnetic torque on the neutron star. The neutron star spins down because the electromagnetic torque generated at the surface of the polar cap opposes its rotation. Inside the neutron star surface, this current flows horizontally towards the edge of the polar cap, where it flows out in a current sheet along the interface between open and closed field lines. The electric current flowing through the polar cap is (to a good approximation) distributed as $I \propto \Psi(2 - \Psi/\Psi_{pc})$, where Ψ is the total amount of magnetic flux contained inside cylindrical radius R , and Ψ_{pc} is the total amount of magnetic flux which opens up to infinity. This is an exact expression for a magnetic (split) monopole, and CKF showed that it remains approximately valid even for a dipole. Since on the neutron star surface $\Psi \propto R^2$ when

$$R < R_{pc} \ll r_0,$$

$$I(R) = I_{pc} \left(\frac{R}{R_{pc}} \right)^2 \left[2 - \left(\frac{R}{R_{pc}} \right)^2 \right]. \quad (2)$$

When this current flows horizontally in a layer of thickness $h(R)$ across the polar cap, an electric current density

$$J(R) = \frac{I_{pc}}{2\pi R h(R)} \left(\frac{R}{R_{pc}} \right)^2 \left[2 - \left(\frac{R}{R_{pc}} \right)^2 \right] \quad (3)$$

will flow horizontally, which, combined with the axial magnetic field B_* threading the polar cap, generates an azimuthal Lorenz force per unit volume, $f(R) = \frac{1}{c} J(R) \times B_0$. Integrating $f(R)$ over the volume of the polar cap crust where the above electric current flows horizontally, and doubling our result to account for the two (north/south) polar caps, we obtain the total electromagnetic torque

$$T \sim \frac{2}{3c} I_{pc} B_0 R_{pc}^2. \quad (4)$$

We present two simple, physically equivalent, estimates of the electric current flowing in the magnetosphere. One is to consider particles (electrons/positrons) with Goldreich–Julian charge densities $\rho_{GJ} \approx B_0/2\pi R_{LC}$ flowing outwards at the speed of light from the polar cap. This naive estimate gives

$$I_{pc} \sim \pi R_{pc}^2 \rho_{GJ} c = \frac{B_0 r_0 c}{2} \left(\frac{r_0}{R_{LC}} \right) \left(\frac{r_0}{r_{\text{open}}} \right) \quad (5)$$

Another equivalent, more physical, way to estimate the total amount of electric current flowing in the magnetosphere through the polar cap is to make the naive (and correct) assumption that, at the distance of the light cylinder the two magnetic field components (toroidal and poloidal) must be of the same order of magnitude, $B_\phi|_{LC} \sim B_p|_{LC}$. This is indeed true in the force-free axisymmetric magnetosphere (without wind), since the light cylinder is the Alfvén point (Li & Melrose 1994). In general, the Alfvén point is some short distance inside the light cylinder. When the two field components are scaled back to the surface of the star at the edge of the polar cap, we obtain

$$B_p|_{pc} = B_p|_{LC} \left(\frac{R_{LC}}{r_{\text{open}}} \right)^2 \left(\frac{r_{\text{open}}}{r_0} \right)^3 \equiv B_0. \quad (6)$$

The structure of the field is dipole-like out to r_{open} and monopole-like out to the light cylinder. From Eqn (6) and the relation $B_\phi|_{LC} \sim B_p|_{LC}$, we have

$$B_\phi|_{pc} = B_0 \frac{r_0^3}{R_{pc} R_{LC} r_{\text{open}}}, \text{ and therefore} \quad (7)$$

$$I_{pc} \sim \frac{B_\phi|_{pc} R_{pc} c}{2} = \frac{B_0 r_0 c}{2} \left(\frac{r_0}{R_{LC}} \right) \left(\frac{r_0}{r_{\text{open}}} \right). \quad (8)$$

This is a very simple result, and shows that the two ways of looking at the problem are equivalent.

Using the above relation for the polar cap current in equation (4) we obtain the expression for the torque T associated with the above spin-down arguments. The corresponding energy loss rate due to the above torque is therefore $\dot{E} = T \cdot \Omega$, given more explicitly by

$$\dot{E}_w = T \cdot \Omega = \frac{B_0^2 r_0^6 \Omega^4}{3 c^3} \left(\frac{R_{LC}}{r_{\text{open}}} \right)^2 = \dot{E}_D \left(\frac{L_p}{\dot{E}_D} \right)^{1/2} \quad (9)$$

where we have used $L_p/4\pi c r_{\text{open}}^2 = B(r_{\text{open}})^2/8\pi$ to obtain an expression for \dot{E}_w in terms of the wind luminosity, L_p , and dipole energy loss, \dot{E}_D . Note that the standard dipole spin-down formula (modulo the different numerical factor in the denominator) is modified by the term $(R_{LC}/r_{\text{open}})^2$ which incorporates the effects of the “loading” of the magnetosphere with the outflowing wind. This expression, while of similar functional form, differs from that of TB98 because of a normalization error in this work (Thompson, priv. comm.), but agrees with a corrected expression given by Thompson et al. (1999).

Using Eqn. (9) with values for $P = P_{1806} = 7.48$ s, $\dot{P} = \dot{P}_{1806} = 8.3 \cdot 10^{-11}$ s s⁻¹, i.e. those observed in SGR1806–20 and assuming the presence of a steady wind of luminosity $L_{37} = L_p/10^{37}$ erg s⁻¹, we obtain an estimate of the surface magnetic field of the neutron star:

$$B_0 \simeq 3 \times 10^{13} \text{G} \left(\frac{P}{P_{1806}} \right)^{-1} \left(\frac{\dot{P}}{\dot{P}_{1806}} \right) L_{37}^{-1/2}, \quad (10)$$

where we have assumed $r_0 = 10$ km and $I = 10^{45}$ g cm². If one uses the observed value of the spin-down rate and the average value of the particle luminosity needed to account for the energetics of the nebula, then B_0 is significantly below the estimate of $B_0 \simeq 10^{15}$ using Eqn (9) with $R_{LC} \simeq r_{\text{open}}$.

This modified spin-down law leads to *exponential* increase in the pulsar period instead of the power law increase associated with the purely dipole emission (this is easily seen from equating $\dot{E}_w = -I\Omega\dot{\Omega}$ and integrating). One can thus estimate the age τ of SGR1806-20 through the relation

$$\tau = \frac{P}{2\dot{P}} \ln \left[\frac{L_p P^3}{4\pi^2 I \dot{P}} \right], \quad (11)$$

which yields $\tau \sim 11,800$ yr for $L_p = 10^{37}$ erg s⁻¹. Thus the steady wind model can naturally account for the fact that the age of the SNR G10.0–0.3 is much larger than the characteristic dipole spin-down age, but with the penalty that the magnetar model must be abandoned.

3. COMBINED WIND AND DIPOLE SPIN-DOWN

The expression for the magnetic field and characteristic age of the neutron star given above are only valid if the relativistic particle wind completely and continuously dominates the spin down of the star. If the particle wind flow is either discontinuous or not dominant, then a more general description of the spin-down energy loss must be used. If the wind has instantaneous luminosity L_p during its times of activity and duty cycle D_p , defined as the fraction of total on-time, then the average energy loss from combined wind and dipole is

$$\begin{aligned}\dot{E} &= -\langle I\Omega\dot{\Omega} \rangle = \dot{E}_D(1 - D_p) + \dot{E}_w D_p \\ &= \frac{B_0^2 r_0^6 \Omega^4}{6c^3} (1 - D_p) + L_p^{1/2} D_p \frac{B_0 r_0^3 \Omega^2}{\sqrt{6c^3}},\end{aligned}\quad (12)$$

where we have used Eqn (9). The surface magnetic field may then be found as the solution to the quadratic equation, giving

$$B_0 = -\frac{\sqrt{6c^3}}{8\pi^2} \frac{L_p^{1/2} D_p P^2}{(1 - D_p) r_0^3} F(P, \dot{P}) \quad (13)$$

where,

$$F(P, \dot{P}) = \left\{ 1 - \left(1 + \frac{4\dot{E}(1 - D_p)}{L_p D_p^2} \right)^{1/2} \right\}, \quad (14)$$

and $\dot{E} = 4\pi^2 I \langle \dot{P} \rangle / P^3$. Note that when $L_p D_p^2 \ll 4\dot{E}(1 - D_p)$,

$$B_0 \simeq \left(\frac{3c^3 I \langle \dot{P} \rangle P}{2\pi^2 r_0^6 (1 - D_p)} \right)^{1/2} \quad (15)$$

which gives the standard dipole formula when $D_p = 0$. If $L_p D_p^2 \gg 4\dot{E}(1 - D_p)$, Eqn (13) gives the pure wind formula Eqn(10) with L_p replaced by $L_p D_p^2$.

We may also integrate Eqn (12) from the initial period P_0 to the present period P to obtain the general expression for the neutron star characteristic age τ . Assuming that $P_0 \ll P$,

$$\tau \simeq -\frac{4\pi^2 I}{L_p D_p^2 P^2} \frac{\ln[1 - 2/F(P, \dot{P})]}{F(P, \dot{P})}. \quad (16)$$

This expression gives the usual characteristic age for dipole spin down, $\tau = P/2\dot{P}$, in the limit $L_p D_p^2 \ll 4\dot{E}(1 - D_p)$ and $D_p = 0$. One must be careful in the limit $L_p D_p^2 \gg 4\dot{E}(1 - D_p)$. If D_p is close to 1, then the first term in Eqn (12) should be dropped to give an expression for τ which is the same as Eqn (11), again with L_p replaced by $L_p D_p^2$.

We now examine the consequences of the general expressions (13) and (16) for SGR1806-20, the SGR source for which we have the best estimate of the particle wind luminosity. Figure 1 shows the values of B_0 and τ computed from Eqns (13) and (16) for the measured P and \dot{P} of SGR 1806-20 (Kouveliotou et al. 1998) as a function of the parameter $L_p D_p^2 / \dot{E}$, indicating the fractional wind contribution to the spin-down energy loss rate, assuming $D_p \ll 1$. For small $L_p D_p^2 / \dot{E}$, the curves approach their dipole radiation values of $B_0 \simeq 10^{15}$ G and $\tau \simeq 1500$ yr. For $L_p D_p^2 / \dot{E} \gtrsim 0.1$, B_0 and τ begin to depart from these values, with the magnetic field decreasing and the age increasing to connect smoothly to the wind-dominated solutions. We have seen in Section II that assuming continuous, wind dominated spin-down can give a characteristic age that agrees with the age ($\tau \sim 10^4$ yr) of the plerion surrounding SGR1806-20, but that the derived magnetic field drops into the range of normal radio pulsars. In such a case, however, the free energy associated with the magnetic field decay is not sufficient to account for the observed luminosity, and an alternative free energy source must be considered.

The parameter $L_p D_p^2$ may be estimated for SGR 1806-20 from its observed burst characteristics. In general, we can write the particle luminosity associated with a burst as $L_p = E_\gamma \epsilon_\gamma^{-1} \Delta\tau_w^{-1}$, where E_γ is the γ -ray burst energy, ϵ_γ is the conversion efficiency of particle energy to γ -rays and $\Delta\tau_w$ is the duration of the wind outflow following the burst. If T is the average time between bursts, then the wind duty cycle is $D_p = \Delta\tau_w / T$, and

$$L_p D_p^2 = E_\gamma \epsilon_\gamma^{-1} \Delta\tau_w T^{-2}. \quad (17)$$

In addition, the requirement that the X-ray nebula (Murakami et al. 1994) is powered by the aggregate of the bursts' wind outflows leads to the condition $L_p D_p = 10^{42} \text{ erg } E_{40} (10^{-2} / \epsilon_\gamma) / T = 10^{37} \text{ erg s}^{-1} (10^{-2} / \eta)$, where η is the conversion efficiency of particle luminosity to nebular emission and $E_{40} \equiv E_\gamma / 10^{40} \text{ erg}$. For the multiple SGR bursts from SGR 1806-20, Eqn (17) and the above requirement on $L_p D_p$, we find $E_{40} (10^{-2} / \epsilon_\gamma) = (T_{SGR} / 10^5 \text{ s}) (10^{-2} / \eta)$, giving $L_p D_p^2 = 10^{34} \text{ erg s}^{-1} (\Delta\tau_w / 10^2 \text{ s}) (10^5 \text{ s} / T_{SGR}) (10^{-2} / \eta)$. Likewise, for the giant bursts, we have $L_p D_p^2 = 3 \times 10^{35} \text{ erg s}^{-1} (\Delta\tau_w / 10^7 \text{ s}) (10 \text{ yr} / T_G) (10^{-2} / \eta)$. From Eqn (13), Eqn (16) and Figure 1, the conflicting goals of preserving the magnetar model (i.e. $B_0 \gtrsim 10^{14}$ G) and bringing the characteristic age within a factor of 2 of the 10^4 yr age of G10.0-0.3, may be satisfied with $L_p D_p^2 / \dot{E} \simeq 10 - 100$. Since $\dot{E} = 8 \times 10^{33} \text{ erg s}^{-1}$, the duration of the particle outflow must be much larger than the γ -ray burst duration and the wind flow duty cycle must be, $D_p \sim 0.008 - 0.08$.

4. DISCUSSION

The results of our analysis leave two alternatives for interpreting the spin down of SGRs, given the present limited data. The first assumes a continuous wind outflow at a luminosity sufficient to yield a characteristic age in agreement with that of the surrounding SNR. We have shown that in the case of SGR1806-20, this assumption leads to a surface magnetic field of only $B_0 = 3 \times 10^{13}$ G, well below the magnetar range ($> 10^{14}$ G). However, this alternative requires a source of free energy other than the field decay to power both the nebular emission and the SGR bursts. The second option assumes an episodic (or at least variable) wind outflow such that the average wind luminosity is sufficient to power the nebular emission. This allows a range of combinations of surface field and age and depends on the wind duty cycle. We show that, in the case of SGR1806-20, it is possible to accommodate both a characteristic age $\tau \sim 7500$ yr, consistent with the estimated SNR age of $\sim 10^4$ yr, and a magnetar model ($B_0 = 10^{14}$ G). One should then observe a sudden increase in the period derivative following SGR bursts. Evidence for such an increase was seen following the bursting activity of June - August 1998 from SGR1900+14 (Marsden et al. 1999). These options assume that gravitational radiation did not play a significant role in early spin-down evolution of the star, but it is unlikely to have made a large difference in the characteristic age.

There are a number of arguments in favor of the magnetar model for SGRs, most of which have been discussed by Thompson & Duncan (1995) and Baring & Harding (1998). An additional argument is that the pure dipole fields of AXPs, which are spinning down smoothly (Gotthelf et al. 1999) and have much lower luminosity wind flows (if any), lie in the magnetar range. We suggest that detailed monitoring of the spin periods of the SGRs, to search for variations in the period derivative, can measure or place limits on the duty cycle of particle outflows and thus determine whether a magnetar model for these sources is viable.

We thank Rob Duncan, Chris Thompson, Peter Woods and the referee Cole Miller for valuable comments and discussions.

REFERENCES

- Baring, M. G. & Harding, A. K. 1998, *ApJ*, 507, L55.
 Contopoulos, I., Kazanas, D. & Fendt, C., 1999, *ApJ*, 511, 351 (CKF).
 Duncan, R. C. & Thompson, C. 1992, *ApJ*, 392, L9.

- Frail, D. A., Vasisht, G. & Kulkarni, S. R 1997, ApJ, 480 1129.
- Goldreich, P. & Julian, W. H., 1969, ApJ, 157, 869.
- Gotthelf, E. V. & Vasisht, G. 1998, New Astronomy, 3. 293.
- Gotthelf, E. V., Vasisht, G. & Dotani, T. 1999 (astro-ph/9906122).
- Harding, A. K. 1995, in High Velocity Neutron Stars and Gamma-Ray Bursts, ed. R. E. Rothschild & R. E. Lingenfelter (AIP Conf. Proc. 366), p. 118.
- Hurley, K. *et al.*, 1999a, ApJ, 510, L111.
- Hurley, K. *et al.*, 1999b, ApJ Letters, in press (astro-ph/9906020).
- Kouveliotou, C. *et al.*, 1998, Nature, 393, 235
- Kouveliotou, C. *et al.*, 1999, ApJ, 510, L115.
- Kulkarni, S. R. *et al.* 1994, Nature, 368, 129.
- Li, J. & Melrose, D. B., 1994, MNRAS, 270, 687.
- Marsden, D., Rothschild, R. E. & Lingenfelter, R. E. 1999, ApJ, 520, L107.
- Murakami, T. *et al.* 1994, Nature, 368, 127.
- Paczynski, B. 1992, Acta Astron., 42, 145.
- Tavani, M. 1994, ApJ, 431, L83.
- Thompson, C. & Blaes, O. 1998, Phys. Rev. D, 57, 3219 [TB98].
- Thompson, C. & Duncan, R. C. 1995, MNRAS, 275, 255.
- Thompson, C. & Duncan R. C., 1996, ApJ, 473, 322.
- Thompson, C. *et al.* 1999 (astro-ph/9908086).
- Usov, V. V. 1992, Nature, 357, 472.

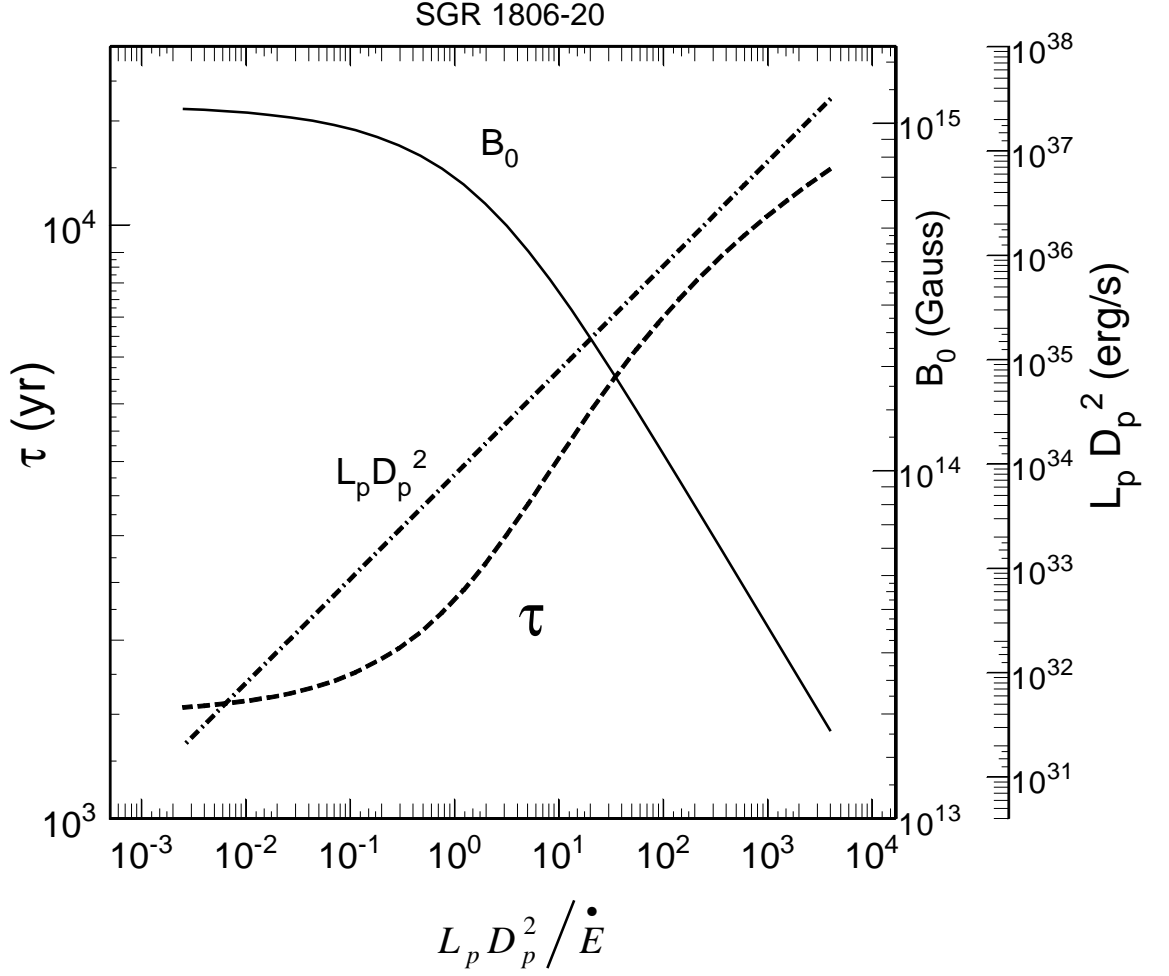


Fig. 1.— Surface dipole magnetic field B_0 and characteristic age τ as a function of the parameter $L_p D_p^2 / \dot{E}$, from the general expressions Eqns (13) and (16) derived from the combined dipole and wind spin-down formula, Eqn (12).